| Facts about Trigonometric Integrals <br> involving Sine and/or Cosine | Explanation |
| :---: | :--- |
| Power Notation for Trig. Functions | We usually use the shorthand notation of <br> $s^{m}(x)$ to mean $(\sin (x))^{m}$. Note that <br> $s^{m} n^{-1}(x)$ denotes the inverse of the sine <br> function and not $(\sin (x))^{-1}$. The above <br> two notation rules for sine also hold for <br> the other trigonometric functions. |
| $\sin ^{2}(\theta)+\cos ^{2}(\theta)=1$ | Pythagorean Identity used to solve <br> Trigonometric Integrals involving odd <br> powers of sine or cosine. |
| $\sin ^{2}(\theta)=\frac{1-\cos (2 \theta)}{2}$ | Half-angle (power reducing) formula for <br> sine used to solve Trigonometric Integrals <br> involving even powers of sine. |
| $\cos ^{2}(\theta)=\frac{1+\cos (2 \theta)}{2}$ | Half-angle (power reducing) formula for <br> cosine used to solve Trigonometric <br> Integrals involving even powers of cosine. |
| Rules to Integrate Products of Powers of <br> Sine and Cosine $($ These rules also work <br> when there is only $\sin (x)$ and $\cos (x)$ in <br> the integrand. That is, $m$ and $n \operatorname{can}$ be <br> zero) | Case 1: If $m$ and $n$ are both odd, then <br> chose only one function (either sine or <br> cosine) to "break one off" and then use <br> the Pythagorean Identity on the <br> remaining even power function. Ignore <br> the power of the other function. <br> Case 2: If $m$ and $n$ do not have the same |
| $\int \sin ^{m}(x) \cos { }^{n}(x) d x$ | parity (one is even and the either is odd $),$ <br> then choose the function with an odd <br> power to "break one off" and then use the <br> Pythagorean Identity on the remaining <br> even power function. Ignore the original <br> even power function. <br> Case 3: If $m$ and $n$ are both even, then <br> use the half-angle identities on both sine <br> and cosine. |

1. Evaluate $\int \sin ^{3}(x) d x$.
2. Evaluate $\int \sin ^{6}(x) \cos ^{3}(x) d x$.
3. Evaluate $\int \sin ^{2}(x) \cos ^{4}(x) d x$.
$\left.\begin{array}{|l|l|}\hline \begin{array}{l}\text { Facts about Trigonometric Integrals } \\ \text { involving Tangent and Secant }\end{array} & \text { Explanation } \\ \hline 1+\tan ^{2}(x)=\sec ^{2}(x) & \begin{array}{l}\text { Pythagorean Identity used to solve } \\ \text { Trigonometric Integrals involving powers } \\ \text { of tangent and secant. }\end{array} \\ \hline \begin{array}{l}\text { Rules to Integrate Products of Powers of } \\ \text { Tangent and Secant }\end{array} & \begin{array}{l}\text { Case 1: If } n \text { (the power of secant) is even, } \\ \text { then break off } \sec ^{2}(x) \text { and use the } \\ \text { Pythagorean Identity } \\ \left(\sec ^{2}(x)=1+\tan ^{2}(x)\right) \text { on the remaining } \\ \text { even power of secant. } \\ \text { Case 2: If } m(\text { the power of tangent }) \text { is }\end{array} \\ \text { odd, break off one } \sec (x) \text { and tan }(x) \\ (\sec (x) \tan (x)), \text { then use the Pythagorean } \\ \text { Identity }\left(\tan ^{2}(x)=\sec ^{2}(x)-1\right) \text { on the } \\ \text { remaining even power of tangent. }\end{array}\right\}$
4. Evaluate $\int \tan ^{6}(x) \sec ^{4}(x) d x$.
5. Evaluate $\int \tan ^{5}(x) \sec ^{9}(x) d x$.
